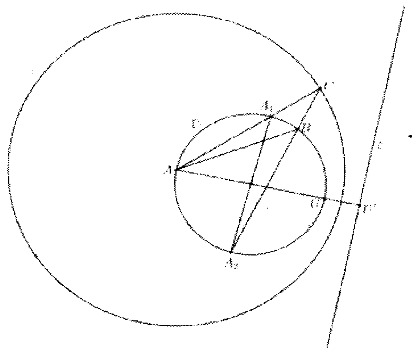
As a result, the maximal value of $\frac{MA}{MC}$ is attained at $J(A_2)$. But $J(A_2) = B$, so we are done.



**W21. (Solution by the proposer.)** The identity

$$2\left(b^2+c^2\right) = 4m_a^2 + a^2 \quad (1)$$

holds. Taking $AM - GM$ inequality, we have

$$4m_a^2 + a^2 = \frac{4}{3}m_a^2 + \frac{4}{3}m_a^2 + \frac{4}{3}m_a^2 + a^2 \geq 4\sqrt[4]{\frac{64}{27}m_a^6a^2}$$

with equality if and only if $\frac{4}{3}m_a^2 = a^2$, from where

$$4m_a^2 + a^2 \geq \frac{8\sqrt[4]{12}m_a\sqrt{am_a}}{3} \quad (2)$$

From (1) and (2) we obtain the inequality

$$b^2 + c^2 \geq \frac{4\sqrt[4]{12}m_a\sqrt{am_a}}{3} \quad (3)$$

with equality if and only if $b^2 + c^2 = 2a^2$. Taking (3) into account the inequality from hypothesis's follows. The equality holds if and only if ABC is equilateral triangle.

**Second solution.** By replacing $(a, b, c, m_a, m_b, m_c)$ in original inequality with $\left(\frac{2m_a}{3}, \frac{2m_b}{3}, \frac{2m_c}{3}, \frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$ (Median Transformation) we obtain equivalent inequality

$$\sum_{cyc}\frac{\left(\left(\frac{2m_b}{3}\right)^2+\left(\frac{2m_c}{3}\right)^2\right)^2}{\left(\frac{a}{2}\right)^3}\geq\frac{32\sqrt{3}}{9}\left(\frac{2m_a}{3}+\frac{2m_b}{3}+\frac{2m_c}{3}\right)\Longleftrightarrow$$

$$\Longleftrightarrow\sum_{cyc}\frac{\frac{16}{81}\left(m_b^2+m_c^2\right)^2}{\frac{a^3}{8}}\geq\frac{32\sqrt{3}}{9}\cdot\frac{2}{3}\left(m_a+m_b+m_c\right)\Longleftrightarrow$$

$$\Longleftrightarrow\sum_{cyc}\frac{\left(m_b^2+m_c^2\right)^2}{a^3}\geq\frac{3\sqrt{3}}{2}\left(m_a+m_b+m_c\right)\Longleftrightarrow$$

$$\Longleftrightarrow\sum_{cyc}\frac{\left(4a^2+b^2+c^2\right)^2}{a^3}\geq 24\sqrt{3}\left(m_a+m_b+m_c\right)\tag{1}$$

(since $m_b^2+m_c^2=\frac{4a^2+b^2+c^2}{4}$)

Applying inequality $\frac{u^2}{v}\geq 2u-v,\ v>0$ for $u=4a^2+b^2+c^2$ and $v=6a^2$ we obtain

$$\frac{\left(4a^2+b^2+c^2\right)^2}{a^3}=\frac{6}{a}\cdot\frac{\left(4a^2+b^2+c^2\right)^2}{6a^2}\geq$$

$$\geq\frac{6\left(2\left(4a^2+b^2+c^2\right)-6a^2\right)}{a}=\frac{12\left(a^2+b^2+c^2\right)}{a}.$$

Therefore,

$$\sum_{cyc}\frac{\left(4a^2+b^2+c^2\right)^2}{a^3}\geq 12\left(a^2+b^2+c^2\right)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right).$$

Since

$$m_a+m_b+m_c\leq\sqrt{3\left(m_a^2+m_b^2+m_c^2\right)}=\sqrt{3\cdot\frac{3}{4}\left(a^2+b^2+c^2\right)}=$$

$$=\frac{3}{2}\sqrt{a^2+b^2+c^2}$$

then

$$24\sqrt{3}\left(m_a+m_b+m_c\right)\le 24\sqrt{3}\cdot\frac{3}{2}\sqrt{a^2+b^2+c^2}=36\sqrt{3}\sqrt{a^2+b^2+c^2}.$$

Thus, suffices to prove inequality

$$12\left(a^2+b^2+c^2\right)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\ge 36\sqrt{3}\sqrt{a^2+b^2+c^2}\iff$$

$$\iff\sqrt{a^2+b^2+c^2}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\ge 3\sqrt{3}\iff$$

$$\iff\left(a^2+b^2+c^2\right)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)^2\ge 27.$$

Since by AM-GM Inequality $a^2+b^2+c^2\ge 3\sqrt[3]{a^2b^2c^2}$ and $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\ge\frac{3}{\sqrt[3]{abc}}$ then $\left(a^2+b^2+c^2\right)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)^2\ge 27$ and, therefore, we have

$$\sum_{cyc}\frac{\left(4a^2+b^2+c^2\right)^2}{a^3}\ge 12\left(a^2+b^2+c^2\right)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\ge$$

$$\ge 36\sqrt{3}\sqrt{a^2+b^2+c^2}\ge 24\sqrt{3}\left(m_a+m_b+m_c\right).$$

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**W22. (Solution by the proposer.)** If we note $\frac{AO}{OC}=k$, then $\frac{AO}{AC}=\frac{k}{k+1}$ and $\frac{CO}{AC}=\frac{1}{k+1}\cdot BCD$. From $\frac{AO}{AC}=\frac{OM}{CB}=\frac{OQ}{CD}$ we have

$$OM=\frac{k}{k+1}BC \qquad (1)$$

$$OQ=\frac{k}{k+1}CD \qquad (2)$$

and from $\frac{CO}{CA}=\frac{ON}{AB}=\frac{OP}{AD}$ we have

$$ON=\frac{1}{k+1}AB \qquad (3)$$

and

$$OP=\frac{1}{k+1}AD \qquad (4)$$